DSGRN Group Meeting Presentation 2

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Computing linear extensions for Boolean lattices with algebraic constraints

I've been going through the recently submitted paper:
"Computing linear extensions for Boolean lattices with algebraic constraints" (Shane Kepley, Konstantin Mischaikow, Lun Zhang)

In the paper, algorithms are given that can solve this problem:
Given a set of linear polynomials $\{p_0, \ldots, p_K\}$, an evaluation domain Ξ that is a polytope, and a partial order $\prec$ such that $p \prec q$ when $p(\xi) < q(\xi)$ for all $\xi \in \Xi$, then what are all the total orders $p_{\sigma(1)} \prec_\sigma \cdots \prec_\sigma p_{\sigma(K)}$ that are a *linear extension* to $\prec$ such that there is some $\xi \in \Xi$ where $p_{\sigma(1)}(\xi) < \cdots < p_{\sigma(K)}(\xi)$?

Parameter Space Decomposition Problem

Given an interaction function:

Definition 1.1. For $n \in \mathbb{N}$, an *interaction function* of order n is a polynomial in n variables, $z = (z_1, \ldots, z_n)$, of the form

$$f(z) = \prod_{j=1}^{q} f_j(z) \tag{4}$$

where each factor has the form

$$f_j(z) = \sum_{i \in I_j} z_i$$

Say that each z_i can either evaluate to ℓ_i or $\ell_i + \delta_i$. This gives 2^n polynomials of the variables $\ell_1, \ldots \ell_n, \delta_1, \ldots, \delta_n > 0$. Which total orders are *admissible*: there is some set of values for our variables that such that these polynomials respect that order.

We can use the algorithms as mentioned in the previous slide to get a stricter set of candidates of the admissible linear orders when we look at interaction functions. With some other techniques, this set can be made even stricter.

Consequences to DSGRN

This gives a major computational improvement:

- Before (2016): DSGRN can handle regulatory networks with nodes that have at most three in-edges and three out-edges.
- Now: DSGRN can handle regulatory networks with nodes that have at most five in-edges (and for some cases of six) and any number of out-edges.

My Goal: To see how well these results can be generalized to handle more values i.e. instead of z evaluating to either ℓ or $\ell + \delta$, what about ℓ, $\ell + \delta_1$, or $\ell + \delta_1 + \delta_2$? Or for an arbitrary number of δ's?